**MATHEMATICS METHODS**

**MAWA Semester 2 (Unit 3&4) Examination 2018**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 1 of term 4, Fri October 12th 2018**

**Section Two: Calculator-assumed (100 Marks)**

**Question 8 (a) (2 marks)**

|  |
| --- |
| Solution |
| Bernoulli distribution with parameter  |
| Mathematical behaviours | Marks |
| * states Bernoulli distribution
* states parameter of

   | 11 |

**Question 8 (b) (2 marks)**

|  |
| --- |
| Solution |
| Variance(*X*)   |
| Mathematical behaviours | Marks |
| * states correct mean
* states variance
 | 11 |

**Question 9 (a) (2 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * determines the expected value
* determines the variance
 | 11 |

**Question 9 (b)**  **(2 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * determines numerator
* determines denominator
 | 11 |

**Question 9 (c) (3 marks)**

|  |
| --- |
| Solution |
|   *F*(*x*) = |
| Mathematical behaviours | Marks |
| * identifies need to integrate *f*(*x*)
* determines definite integral using correct limits of integration (2,*x*)
* determines *F(x)* and states it as a piecewise function
 | 111 |

**Question 9 (d) (2 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * states
* solves for *x*
 | 11 |

**Question 10 (a) (2 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * substitutes and to obtain required equation
* solves equation to obtain
 | 11 |

**Question 10 (b) (2 marks)**

|  |
| --- |
| Solution |
| In 15 years the country’s population =  In 15 years the city’s population = where represents its growth rateHence =Solving gives Hence the continuous growth rate is approximately 7%. |
| Mathematical behaviours | Marks |
| * equates city’s population to 40% of country’s population in 15 years
* solves equation and states percentage growth rate
 | 11 |

**Question 11 (6 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * indicates an appropriate expression involving an integral to determine

 required area* indicates *x* axis values, ln 2 and ln 3
* determines A1
* substitutes correct bounds to determine A2
* evaluates integral and A2
* rearranges expression using log laws and simplifies
 | 111111 |

**Question 12 (a) (1 mark)**

|  |
| --- |
| Solution |
| Confidence interval is $\left(\hat{p}-E,\hat{p}+E\right)=\left(0.53,0.61\right)$So $\hat{p}=\frac{0.53+0.61}{2}=0.57$ |
| Mathematical behaviours | Marks |
| * Obtains correct answer
 | 1 |

**Question 12 (b) (2 marks)**

|  |
| --- |
| Solution |
|  $E=z\_{α}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ie $0.04=1.96\sqrt{\frac{0.57×0.43}{n}}$ Solving for $n$ gives $n≅588.5$So the sample size was 589 (approximately) |
| Mathematical behaviours | Marks |
| * Uses
* Solves for $n$ and rounds
 | 11 |

**Question 12 (c) (3 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * Substitutesvalues into error equation
* Solves for $z\_{α}$
* States level of confidence to at least 1 decimal place
 | 111 |

**Question 12 (d) (2 marks)**

|  |
| --- |
| Solution |
| From the confidence interval in 10(c), there is a 99.94% probability that $p$ lies between 0.5 and 0.64, and in particular $p>0.5.$ So the claim is justified. |
| Mathematical behaviours | Marks |
| * States the claim is justified
* Gives a valid reason
 | 11 |

**Question 13 (a) (3 marks)**

|  |
| --- |
| Solution |
|  Since velocity and acceleration are opposing one another, the particle is slowing down |
| Mathematical behaviours | Marks |
| * calculates
* differentiates  to obtain  and
* states particle is slowing down
 | 111 |

**Question 13 (b) (2 marks)**

|  |
| --- |
| Solution |
| Hence its final position is 144m from the origin. |
| Mathematical behaviours | Marks |
| * integrates velocity equation to determine displacement equation including

  * substitutes , calculates and states final position, with unit
 | 11 |

**Question 14 (4 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * states
* differentiates
* solves
* states co-ordinates of
 | 1111 |

**Question 15 (a) (1 mark)**

|  |
| --- |
| Solution |
| Probability of winning a prize = 0.1 + 0.001 = 0.101 |
| Mathematical behaviours | Marks |
| * uses Addition Principle to calculate the correct probability
 | 1 |

**Question 15 (b) (2 marks)**

|  |
| --- |
| Solution |
| *X*$\~Bin(20,0.101)$ |
| Mathematical behaviours | Marks |
| * states Binomial distribution
* states correct parameters
 | 11 |

**Question 15 (c) (2 marks)**

|  |
| --- |
| Solution |
|  = $\left(\begin{matrix}20\\0\end{matrix}\right)\left(0.101\right)^{0}\left(0.899\right)^{20}+\left(\begin{matrix}20\\1\end{matrix}\right)\left(0.101\right)^{1}\left(0.899\right)^{19}+\left(\begin{matrix}20\\2\end{matrix}\right)\left(0.101\right)^{2}\left(0.899\right)^{18}+\left(\begin{matrix}20\\3\end{matrix}\right)\left(0.101\right)^{3}\left(0.899\right)^{17}$ |
| Mathematical behaviours | Marks |
| * identifies that “no more than 3 prizes” means “can win 0,1,2, or 3 prizes”
* states correct expression for the probability
 | 11 |

**Question 15 (d) (1 mark)**

|  |
| --- |
| Solution |
|  Bincdf(0,3,20,0.101)=0.8634 |
| Mathematical behaviours | Marks |
| * states probability
 | 1 |

**Question 15 (e) (1 mark)**

|  |
| --- |
| Solution |
| Using CAS,*k*=3 |
| Mathematical behaviours | Marks |
| * states *k*=3
 | 1 |

**Question 16 (a) (i) (2 marks)**

|  |
| --- |
| Solution |
| = -3 + 4 – 4 = -3 |
| Mathematical behaviours | Marks |
| * indicates addition of signed areas
* determines result
 | 11 |

**Question 16 (a) (ii) (2 marks)**

|  |
| --- |
| Solution |
| Area = 3+4+4=11  |
| Mathematical behaviours | Marks |
| * expresses the area as a sum of areas
* determines result
 | 11 |

**Question 16 (b) (3 marks)**

|  |
| --- |
| Solution |
| $ $ |
| Mathematical behaviours | Marks |
| * applies the additivity of integrals to split the integral
* applies the linearity of integrals to deduce
* determines result
 | 111 |

**Question 16 (c) (3 marks)**

|  |
| --- |
| Solution |
| Maximum value of  occurs where    Hence, max is 1.  |
| Mathematical behaviours | Marks |
| * applies the Fundamental Theorem
*
* determines maximum value
 | 111 |

**Question 17 (a) (1 mark)**

|  |
| --- |
| Solution |
|

|  |  |  |  |
| --- | --- | --- | --- |
|  | small | medium | large |
| Proportion of peaches |  |  | 0.1538 |

 |
| Mathematical behaviours | Marks |
| * determines both probabilities
 | 1 |

**Question 17 (b) (3 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * states a calculation to determine the mean or variance
* determines mean
* determines standard deviation
 | 111 |

**Question 17 (c) (3 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * states linear transformation required
* determines mean
* determines standard deviation
 | 111 |

**Question 18 (a) (4 marks)**

|  |
| --- |
| Solution |
|  For stationary points,  Thus exact coordinates of minimum turning point =  |
| Mathematical behaviours | Marks |
| * differentiates correctly using the product rule
* equates first derivative to zero to determine *x* co-ordinate of stationary

 points* uses second derivative test to determine nature of turning point
* determines correct coordinates of turning point
 | 1111 |

**Question 18 (b) (1 mark)**

|  |
| --- |
| Solution |
| Since  |
| Mathematical behaviours | Marks |
| * states second derivative is never zero hence no P.O.I.
 | 1 |

**Question 19 (a) (2 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * substitutes
* solves the equation
 | 11 |

**Question 19 (b) (2 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * rearranges log expression correctly
* substitutes  and states  in terms of
 | 11 |

**Question 19 (c) (1 mark)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * evaluates the ratio correctly
 | 1 |

**Question 20 (a) (6 marks)**

|  |
| --- |
| Solution |
|  $ \frac{dy}{dt}=-abe^{-bt}\sin(ct)+ace^{-bt}\cos(ct)=ae^{-bt}\left(c\cos(ct-b\sin(ct))\right)$     |
| Mathematical behaviours | Marks |
| * uses (9,0) to conclude
* solves for $c$
* determines derivative function
* equates derivative function to 0 at  and solves for $b$
* uses (4,60) to solve for $a$
* gives all answers correct to 3 significant figures
 | 111111 |

**Question 20 (b) (3 marks)**

|  |
| --- |
| Solution |
| $y\left(t+\frac{π}{c}\right)=ae^{-b\left(t+\frac{π}{c}\right)}\sin(\left(c\left(t+\frac{π}{c}\right)\right))$ $=ae^{-bt}e^{-\frac{bπ}{c}}\sin(\left(ct+π\right)) $ $=-ae^{-bt}e^{-\frac{bπ}{c}}\sin(ct)=-ry\left(t\right)$  |
| Mathematical behaviours | Marks |
| * replaces  with  to obtain required function
* uses indices laws to factor out
* uses  to complete argument
 | 111 |

**Question 20 (c) (3 marks)**

|  |
| --- |
| Solution |
| The mass travels 120 cm in the first 9 seconds.In the next 9-second period the mass travels $120r$ cm, where from part (b) $r=e^{-πb/c}≅e^{-π×\frac{0.0616}{0.349}}≅0.574$ Hence distance travelled between its first and second return to the origin is   |
| Mathematical behaviours | Marks |
| * deduces that the mass travels 120 m in the first 9 seconds
* evaluates *r*
* obtains correct answer to the nearest centimetre
 | 111 |

**Question 21 (a) (3 marks)**

|  |
| --- |
| Solution |
|  So the 95% confidence interval is $0.629<p<0.699$ |
| Mathematical behaviours | Marks |
| * calculates sample proportion correctly
* calculate standard error correctly
* calculates interval correctly
 | 111 |

**Question 21 (b) (2 marks)**

|  |
| --- |
| Solution |
| Because the old satisfaction rate (65%) lies within the new confidence interval, the recent survey does not provide conclusive evidence that the satisfaction rate has improved.  |
| Mathematical behaviours | Marks |
| * states that survey is not conclusive
* states a valid reason
 | 11 |

**Question 21 (c) (i) (3 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * substitutes
* expresses error as a constant
* deduces error
 | 111 |

**Question 21 (c) (ii) (1 mark)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * states Margin of error
 | 1 |

**Question 21 (c) (iii) (1 mark)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * uses the relationship between the errors to draw valid conclusion
 | 1 |

**Question 21 (d) (3 marks)**

|  |
| --- |
| Solution |
| True margin of error  |
| Mathematical behaviours | Marks |
| * substitute and equates true  to  for simple interval
* solves inequality to determine
* states 97% level of confidence
 | 111 |

**Question 21 (e) (1 mark)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * calculates  and deduces that sample size is appropriate
 | 1 |