**MATHEMATICS METHODS**

**MAWA Semester 2 (Unit 3&4) Examination 2018**

**Calculator-assumed**

# Marking Key

© MAWA, 2018

**Licence Agreement**

This examination is Copyright but may be freely used within the school that purchases this licence.

* The items that are contained in this examination are to be used solely in the school for which they are purchased.
* They are not to be shared in any manner with a school which has not purchased their own licence.
* The items and the solutions/marking keys are to be kept confidentially and not copied or made available to anyone who is not a teacher at the school. Teachers may give feedback to students in the form of showing them how the work is marked but students are not to retain a copy of the paper or marking guide until the agreed release date stipulated in the purchasing agreement/licence.

The release date for this exam and marking scheme is

* **the end of week 1 of term 4, Fri October 12th 2018**

**Section Two: Calculator-assumed (100 Marks)**

**Question 8 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Bernoulli distribution with parameter | |
| Mathematical behaviours | Marks |
| * states Bernoulli distribution * states parameter of | 1  1 |

**Question 8 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Variance(*X*) | |
| Mathematical behaviours | Marks |
| * states correct mean * states variance | 1  1 |

**Question 9 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines the expected value * determines the variance | 1  1 |

**Question 9 (b)**  **(2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines numerator * determines denominator | 1  1 |

**Question 9 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| *F*(*x*) = | |
| Mathematical behaviours | Marks |
| * identifies need to integrate *f*(*x*) * determines definite integral using correct limits of integration (2,*x*) * determines *F(x)* and states it as a piecewise function | 1  1  1 |

**Question 9 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states * solves for *x* | 1  1 |

**Question 10 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * substitutes and to obtain required equation * solves equation to obtain | 1  1 |

**Question 10 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| In 15 years the country’s population =  In 15 years the city’s population = where represents its growth rate  Hence =  Solving gives  Hence the continuous growth rate is approximately 7%. | |
| Mathematical behaviours | Marks |
| * equates city’s population to 40% of country’s population in 15 years * solves equation and states percentage growth rate | 1  1 |

**Question 11 (6 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * indicates an appropriate expression involving an integral to determine   required area   * indicates *x* axis values, ln 2 and ln 3 * determines A1 * substitutes correct bounds to determine A2 * evaluates integral and A2 * rearranges expression using log laws and simplifies | 1  1  1  1  1  1 |

**Question 12 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Confidence interval is  So | |
| Mathematical behaviours | Marks |
| * Obtains correct answer | 1 |

**Question 12 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| ie  Solving for gives  So the sample size was 589 (approximately) | |
| Mathematical behaviours | Marks |
| * Uses * Solves for and rounds | 1  1 |

**Question 12 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Substitutesvalues into error equation * Solves for * States level of confidence to at least 1 decimal place | 1  1  1 |

**Question 12 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| From the confidence interval in 10(c), there is a 99.94% probability that lies between 0.5 and 0.64, and in particular So the claim is justified. | |
| Mathematical behaviours | Marks |
| * States the claim is justified * Gives a valid reason | 1  1 |

**Question 13 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since velocity and acceleration are opposing one another, the particle is slowing down | |
| Mathematical behaviours | Marks |
| * calculates * differentiates  to obtain  and * states particle is slowing down | 1  1  1 |

**Question 13 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence its final position is 144m from the origin. | |
| Mathematical behaviours | Marks |
| * integrates velocity equation to determine displacement equation including      * substitutes , calculates and states final position, with unit | 1  1 |

**Question 14 (4 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states * differentiates * solves * states co-ordinates of | 1  1  1  1 |

**Question 15 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Probability of winning a prize = 0.1 + 0.001  = 0.101 | |
| Mathematical behaviours | Marks |
| * uses Addition Principle to calculate the correct probability | 1 |

**Question 15 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| *X* | |
| Mathematical behaviours | Marks |
| * states Binomial distribution * states correct parameters | 1  1 |

**Question 15 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| = | |
| Mathematical behaviours | Marks |
| * identifies that “no more than 3 prizes” means “can win 0,1,2, or 3 prizes” * states correct expression for the probability | 1  1 |

**Question 15 (d) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Bincdf(0,3,20,0.101)=0.8634 | |
| Mathematical behaviours | Marks |
| * states probability | 1 |

**Question 15 (e) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Using CAS,  *k*=3 | |
| Mathematical behaviours | Marks |
| * states *k*=3 | 1 |

**Question 16 (a) (i) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| = -3 + 4 – 4 = -3 | |
| Mathematical behaviours | Marks |
| * indicates addition of signed areas * determines result | 1  1 |

**Question 16 (a) (ii) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Area = 3+4+4=11 | |
| Mathematical behaviours | Marks |
| * expresses the area as a sum of areas * determines result | 1  1 |

**Question 16 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * applies the additivity of integrals to split the integral * applies the linearity of integrals to deduce * determines result | 1  1  1 |

**Question 16 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Maximum value of  occurs where    Hence, max is 1. | |
| Mathematical behaviours | Marks |
| * applies the Fundamental Theorem * determines maximum value | 1  1  1 |

**Question 17 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  |  | | --- | --- | --- | --- | |  | small | medium | large | | Proportion of peaches |  |  | 0.1538 | | |
| Mathematical behaviours | Marks |
| * determines both probabilities | 1 |

**Question 17 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states a calculation to determine the mean or variance * determines mean * determines standard deviation | 1  1  1 |

**Question 17 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states linear transformation required * determines mean * determines standard deviation | 1  1  1 |

**Question 18 (a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| For stationary points,    Thus exact coordinates of minimum turning point = | |
| Mathematical behaviours | Marks |
| * differentiates correctly using the product rule * equates first derivative to zero to determine *x* co-ordinate of stationary   points   * uses second derivative test to determine nature of turning point * determines correct coordinates of turning point | 1  1  1  1 |

**Question 18 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Since | |
| Mathematical behaviours | Marks |
| * states second derivative is never zero hence no P.O.I. | 1 |

**Question 19 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * substitutes * solves the equation | 1  1 |

**Question 19 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * rearranges log expression correctly * substitutes  and states  in terms of | 1  1 |

**Question 19 (c) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * evaluates the ratio correctly | 1 |

**Question 20 (a) (6 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * uses (9,0) to conclude * solves for * determines derivative function * equates derivative function to 0 at  and solves for * uses (4,60) to solve for * gives all answers correct to 3 significant figures | 1  1  1  1  1  1 |

**Question 20 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * replaces  with  to obtain required function * uses indices laws to factor out * uses  to complete argument | 1  1  1 |

**Question 20 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| The mass travels 120 cm in the first 9 seconds.  In the next 9-second period the mass travels cm,  where from part (b)  Hence distance travelled between its first and second return to the origin is | |
| Mathematical behaviours | Marks |
| * deduces that the mass travels 120 m in the first 9 seconds * evaluates *r* * obtains correct answer to the nearest centimetre | 1  1  1 |

**Question 21 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| So the 95% confidence interval is | |
| Mathematical behaviours | Marks |
| * calculates sample proportion correctly * calculate standard error correctly * calculates interval correctly | 1  1  1 |

**Question 21 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Because the old satisfaction rate (65%) lies within the new confidence interval, the recent survey does not provide conclusive evidence that the satisfaction rate has improved. | |
| Mathematical behaviours | Marks |
| * states that survey is not conclusive * states a valid reason | 1  1 |

**Question 21 (c) (i) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * substitutes * expresses error as a constant * deduces error | 1  1  1 |

**Question 21 (c) (ii) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states Margin of error | 1 |

**Question 21 (c) (iii) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * uses the relationship between the errors to draw valid conclusion | 1 |

**Question 21 (d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| True margin of error | |
| Mathematical behaviours | Marks |
| * substitute and equates true  to  for simple interval * solves inequality to determine * states 97% level of confidence | 1  1  1 |

**Question 21 (e) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * calculates  and deduces that sample size is appropriate | 1 |